# LIM-G: Learner-initiating Instruction Model based on Cognitive Knowledge for Geometry Word Problem Comprehension

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**Abstract**

Computer-assisted instruction systems have been broadly applied to help students solve math word problem. The majority of such systems, which are based on an instructor-initiating instruction strategy, provide pre-designed problems for the learners. When learners are asked to solve a word problem, the system will instruct the learners what to do. However, systems employing an instructor-initiating instruction strategy offer little help to advanced learners or to learners encountering problems that are not in the pre-designed database. Therefore, in this study, a Learner-initiating Instruction Model (LIM-G) is proposed to help learners' comprehension of geometry word problems. Geometry word problems are math word problems involving geometric concepts. Many researches indicate that learners encounter difficulties while comprehending math word problems. In this model, a learner can seek help with any geometry word problem he is interested in. Based on a learner-initiating instruction strategy, LIM-G first comprehends the problem and then gives the learner the telegraphic and diagrammatic representations of the problem, which are more intuitive to understand. For LIM-G, the comprehension mechanism plays a critical role in solving word problems. For this study, a system is built based on LIM-G. In

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this system, the cognitive knowledge needed for comprehending geometry word problem is constructed with an ontology-based tool called InfoMap. Using cognitive knowledge and frame-template structures, the system can extract the relevant concepts in geometry word problems for comprehension.

*Keywords*: Intelligent tutoring system, learner-initiating instruction, problem posing, geometry word problem, problem comprehension, problem solving, diagrammatic and telegraphic representation,

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## 1. Introduction

Problem solving plays a very important role in math education. In USA, the National Council for Teachers of Mathematics (NCTM) recommends in its Agenda for Action (NCTM, 1980) that problem solving is a focus of mathematics education. Their Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) also identifies problem solving as the central focus of the mathematics curriculum in elementary schools. Solving math word problems is an important topic at elementary schools because they are used to train children to apply the formal mathematical knowledge and skills learned at school. Besides, math word problems are thought of as a vehicle for developing students' general problem solving capacity and for making the mathematics lessons more pleasant and motivating (De Corte & Verschaffel, 1989; Treffers, 1987). Common math word problem types learned in elementary and secondary schools include arithmetic word problem, algebra word problem, time word problem, and geometry word problem, etc. Previous researches have shown that many students have trouble in solving math word problems. These problems are difficult for students because both language proficiency and mathematical knowledge are needed for solving them (Cummins, Kintsch, Reusser, & Weimer, 1988; Cummins, 1991; Davis-Dorsey, Ross, & Morrison, 1991). Therefore, many computer-assisted instruction systems are developed to help students learn how to solve various types of math word problems (e.g., Chang, Song, & Lin, 2005; Koedinger & Sueker, 1996; Looi

& Tan, 1998; Steele & Steele, 1999; Wheeler & Regian, 1999).

The Pump Algebra Tutor (PAT) (Koedinger, Anderson, Hadley, & Mark, 1997; Koedinger & Sueker, 1996) is designed to help students develop algebra skills which they can use in the context of real-life problem situations. The student solves a word problem by representing the problem in various ways, such as tables and graphs, and by using those representations to understand the problem and answer the questions. Enabling students to understand and use multiple representations of information such as texts, tables, and graphs, is a major focus of the tutor. WORDMATH (Looi & Tan, 1999) uses a model-building method to provide learners with a learning environment for solving arithmetic word problems. The system displays pre-designed problems and their models to the learners. Through the process of model building, the learners would understand these word problems and then solve them. Steele and Steele (1999) describe an intelligent tutoring system, DISCOVERY, which helps students learn how to solve arithmetic word problem. The system displays pre-designed problems and assists learners to do problem solving based on a direct teaching approach. The WPS Tutor (Wheeler & Regian, 1999) is designed to teach algebra word problem solving for ninth grade students. This system is pedagogically based on five cognitive theoretical foundations, including learning by practice, elaboration, categorization, mastery, and induction. Chang, Song, and Lin (2005) propose MathCAL, which is based on Polya's problem-solving stages: comprehending the problem, making a plan, executing the plan, and evaluating the solution. The system assists to achieve a successful outcome at each stage. Schema representation and solution tree are used in the stages of plan making and plan execution.

The instruction process of these systems generally follow these steps: *(1) system presents a problem to learner, (2) learner solves the problem, (3) system evaluates the answer and stops if the answer is acceptable, otherwise do step 4, (4) system provides feedback to learner, and (5) learner revises solution and go to step 3.* All the systems mentioned previously are designed with an instructor-initiating instruction strategy. The learner is passive in receiving pre-designed problems provided by the system and is asked to solve the problems. This strategy works for beginners who develop their competence for problem solving from the very beginning. The question is, nevertheless, such systems might not be helpful if a learner needs help in solving a problem that does not exist in the pre-design database. Consequently, it is definitely not enough for an instruction system to be merely equipped with an instructor-initiating instruction strategy. In order to address this issue, a learner-initiating instruction strategy is needed in an instruction system for more advanced learners. By following the steps of a learner-initiating instruction strategy, a learner becomes active in posing problems that he is interested in. Based on the problem posed by the learner, the system can provide him with relevant teaching materials and hints for problem comprehension and problem solving.

It is necessary for an instruction system with a learner-initiating instruction strategy to be able to comprehend the problems posed by a learner in order to solve these problems. We propose a learner-initiating instruction model (LIM) that comprehends word problems with cognitive knowledge. After comprehending a problem, LIM can extract the relevant concepts in the problem and provide related materials and incremental guidance for the learner to solve the problem. In this study we call the model LIM-G, because it is applied to geometry word problems, which can be considered as a subclass of math word problems. Previous studies indicate that many students have trouble in solving math word problems because they need both language proficiency and mathematical knowledge (Cummins, 1991; Cummins, Kintsch, Reusser, & Weimer, 1988; Davis-Dorsey, Ross, & Morrison, 1991). Furthermore, others (Mayer, 1987; Wu, 1990) also found that students experienced difficulties in understanding math word problems. Focusing on these learners' problems, LIM-G aims at helping elementary school students to comprehend geometry word problems.

In this paper, the difficulties in solving geometry word problems, especially those of problem comprehension, will be described in Section 2. In Section 3, a learner-initiating instruction strategy especially designed for geometry word problems will be explained. The LIM-G architecture and its cognitive knowledge representation will be presented in Section 4. Moreover, we will describe a model of problem concept retrieval, which is the core engine for problem comprehension. The extent to which LIM-G can comprehend geometry word problems is tested with experiments reported in Section 5. The potential applications of LIM-G and future work will be described in Section 6. Conclusive remarks will be made in the last section.

## 2. Geometry word problems

Math word problems are commonly used in elementary and secondary math education for problem solving. They can be categorized into five types according to the domain knowledge involved. They are arithmetic word problem, geometry word problem, time word problem, velocity word problem, and algebra word problem (Table 1). The arithmetic word problem is studied most extensively and is first encountered by students in elementary school. Arithmetic word problems are generally presented in natural language as problems about commonly encountered objects. For example, “ Joe has 8 marbles. Tom has 5 marbles. How many marbles does Tom have less than Tom? ” In order to solve this type of problems, students need to understand the concept of number and when to apply the four fundamental arithmetic operations. Geometry word problems are math word problems about geometric concepts. In order to solve geometry word problems, the learner needs to be familiar with geometric concepts and also skillful in solving arithmetic word problems. Consider this problem: “There is a rectangular garden whose length is 10cm longer than its width. The length is 30cm. Please find the area of the garden.” In order to find the rectangular area, a learner needs to recognize the garden as a rectangle, calculate the width of the rectangular garden, and know that rectangular area is the product of the length and the width.

**< Insert Table 1>**

Math word problems in elementary and junior high schools can be classified according to their subject domains. Some common domains are arithmetic, algebra, time, velocity, and geometry. Among these domains, arithmetic word problems are most extensively studied in the literature. All these domains involve basic operations of addition, subtraction, multiplication and division and sometimes change of units such as from kilometers to meters. Velocity problems generally involve formulae such as velocity * time = distance; geometry problems generally involve formulae such as area of rectangle = width * length of rectangle; the other domains are simpler since learners need not memorize formulae like these. Moreover, geometry problems are more complicated than velocity problems since each geometric shape (e.g., rectangle, circle) has its own formulae of area and perimeter. Thus geometry word problems require more domain knowledge than the other domains. If a learner picks the correct formulae for a geometry problem and makes the correct substitution of numeric values for variables, the remaining problem solving process is the same as that in the simpler domains. Therefore, we believe if the LIM-G model works well for geometry, it can also extend to other domains.

### *2.1 The difficulties for comprehending geometry word problems*

There are relatively few studies on the difficulties learners encounter while solving geometry word problems. Due to the fact that geometry word problems have many characteristics of arithmetic word problems, learners are expected to have similar difficulties when solving both types of problems. In this section, we will discuss the difficulties learners encounter in solving geometry word problems by reviewing literature about learner's difficulties in arithmetic and geometry word problem solving.

Arithmetic word problems play an important role in math education and they are taught in elementary schools at several grades. A lot of researches suggest that many students have trouble in solving arithmetic word problems (e.g., Cummins, Kintsch, Reusser, & Weimer, 1988; Cummins, 1991; Davis-Dorsey, Ross, & Morrison, 1991). Some students perform poorly even in solving very simple word problems. Some researchers found that word problems are difficult for learners because so many skills are involved, such as reading comprehension, equation writing and arithmetic calculation (Cummins, Kintsch, Reusser, & Weimer, 1988; Mayer, 1992; Stern, 1993). Moreover, some studies indicate that students encounter difficulties at the stage of problem comprehension (Mayer, 1987; Wu, 1990).

Indeed, many factors accounting for the difficulties of word problem comprehension have been found through empirical experiments. Some studies show that students fail to solve word problems because they could only recognize the problems' surface information rather than fully comprehend the implications of the problems (Cummins, Kintsch, et al, 1988; Cummins, 1991; Davis-Dorsey, Ross, & Morrison, 1991; Verschaffel, De Corte, & Pauwels, 1992). While dealing with a problem of addition and subtraction, students may misunderstand the problem by interpreting some keywords out of context. For example, a student might take the word *more* in a problem as a signal of addition and the word *less* as a subtraction signal. Consider the following problem: "There is a rectangle whose base is 10cm *longer* than its height. The base is 20cm. Please find the area of this triangle." Some students would see the word *longer* as an addition signal and jump to the incorrect conclusion that the height of the rectangle is 20cm plus 10cm. Some literatures (Tan, 1998; Tai, 2001) indicate that when students solve geometry word problems, they always directly use a formula without comprehending what answer the problem asks for. Consider the problem: "There is a round skating rink with diameter 30m. Please find the area of this rink." When solving this problem, most students will directly apply the formula of circumference and ignore that the problem is to find the area rather than the circumference. In addition, research results indicate that many students fail to comprehend the linguistic descriptions of some key concepts (Cummins, Kintsch, Reusser, & Weimer, 1988; Cummins, 1991). Some researches suggest that other design features of word problems can contribute to comprehension difficulties as well (Dark & Benbow, 1990). For instance, difficult vocabularies or too much irrelevant information in the problems can overwhelm and confuse students. In addition, when students lack the domain knowledge assumed by the word problems, they are also more likely to have difficulties with problem comprehension.

*2.2 Cognitive process of solving math word problems*

According to the cognitive psychologist Mayer (1992), the process of solving math word problems has two steps, problem representation and problem solution. For problem representation, a learner needs to transform a problem's description to his internal mental representation in two stages: problem translation and integration. Problem translation extracts geometric concepts from the textual description of the problem by using linguistic and semantic knowledge. Linguistic knowledge is used to comprehend the words' meanings in the textual description, while semantic knowledge means factual knowledge in the world.

Problem integration requires a learner to connect the sentences in a problems' description and produce a coherent representation. At this stage, schematic knowledge of problem classification is needed to integrate the pieces of information provided by the problem. Moreover, schematic knowledge allows a learner to determine the category of a problem, for example, a rectangular area problem, and to ignore irrelevant information in the problem. Here is an example of geometry word problem that will be used throughout this paper:

*"Mr. Wang bought a rectangular orchard in a small town in central Philadelphia. He measured this land with a ruler and got 1km for the length and 500m for the width. Please find the area of this orchard in square meters."*…………………………………. [Problem 1]

The knowledge for comprehending Problem 1 is listed in Table 2.

**< Insert Table 2>**

After the problem's description is translated into the learner's internal mental representation, it means that the learner have already comprehended the problem. After problem comprehension, the next stage is problem solving. At this stage, the learner will do planning, monitoring, and finally execution to get the answer (Mayer, 1992). Since this paper focuses only on the mechanism of problem comprehension, we do not address the issue of problem solving any further.

### *2.3 Chinese geometry word problems in Taiwan's elementary schools*

Chinese geometry word problems are taught from grade four to grade six in Taiwan's elementary schools. The shapes used in the geometry word problems include squares, rectangles, trapezoids, parallelograms, sectors, and triangles. Table 3 shows the distribution of Chinese geometry word problems in Taiwan's elementary schools. We can find geometry word problems not only in the chapters on geometric concepts but also in the chapters on arithmetic calculation.

**< Insert Table 3>**

In this study, the Chinese geometry word problems in Taiwan's elementary schools are categorized according to three variables: geometric shape, given information, and the target geometric feature. We have analyzed 282 problems from textbooks and reference books and come up with the categories listed in Table 4. There are seven major categories based on the geometric shapes used in the problems. Each major category is further divided into subcategories.

**< Insert Table 4>**

### *2.4 Problem representation of geometry word problem*

So far we have discussed learners' difficulties and their cognitive process for solving geometry word problems in Section 2.1 and 2.2. Previous studies show that many learners encounter difficulties at the stage of problem representation. Many researchers proposed various methods to help students comprehend math word problems (Goldin, 1987; De Corte, Verscjaffel, & De Win, 1985). One robust method is to use different representations to describe the problems (Chen, 2000; Moyer, Sowder, Threadgill-Sowder, & Moyer, 1984; Sowder & Threadgill-Sowder, 1982; Wu, 1990). In this section we discuss how to use different problem representations to help students understand geometry word problems.

In previous studies, researchers have used three major representational forms for presenting word problems: textual, diagrammatic, and telegraphic representations. Experimental results indicate that different forms of representations influence problem-solving performances. Sowder and Threadgill-Sowder (1982) compared the performance of 262 fifth grade students in solving word problems that are displayed with textual and diagrammatic problem representations. They found that the students performed significantly better on problems with diagrammatic representation than those with textual representation. Moyer, Sowder, Threadgill-Sowder, and Moyer (1984) divided 854 third-to-seventh grade students into two groups according to their levels of reading ability. Both groups concluded that diagrammatic representation is more helpful than textual representation for comprehending word problems. As for problems with telegraphic and textual representations, their effects on the students' comprehension had no significant difference.

There are also researches in Taiwan on problem representations. Wu (1990) compared 146 fifth grade elementary school students in solving multi-step math word problems with diagrammatic representation and textual representation. He found that the students improved their accuracy of problem solving with diagrammatic representation. Chen (2000) studied the performance of 133 fifth grade elementary school students in solving problems with textual, telegraphic, and diagrammatic representations. The result suggested that their performance on problems with diagrammatic representation was the best, followed by problems with telegraphic representation and then by textual problems.

Form the above researches we find that providing different problem representations can help the learner comprehend word problems. Therefore, in this study we assist learners' comprehension of geometry word problems by providing both diagrammatic and telegraphic representations. Table 5 uses an example to illustrate the three representations of a geometry word problem.

**< Insert Table 5>**

3. **Learner-initiating instruction strategy for geometry word problems**

In this study, the learner-initiating instruction strategy follows four steps: comprehend a given problem, show problem representations, diagnose comprehension level, and highlight the features in question. First of all, upon accepting a problem from a learner, the system determines the category of the problem and produces the problem's concept-attribute content module based on cognitive knowledge, which will be discussed in the next section. Then the system provides diagrammatic and telegraphic representations to help the learner comprehend the problem. Moreover, the system asks the learner some questions to diagnose how well the learner comprehends the problem. The diagnosis questions are generated according to the problem's concept-attribute content. Finally, if the learner's comprehension is not satisfactory, the system assists the learner to comprehend the problem by highlighting the key features in the telegraphic and diagrammatic representations. In the next section, the cognitive knowledge structure for comprehending geometry word problems will be introduced.

*3.1 Cognitive knowledge structure for comprehending geometry word problems*

In this paper, cognitive knowledge means the knowledge needed for comprehending geometry word problems. The cognitive knowledge structure we use is a tree, which contains hierarchical nodes of schematic knowledge, problem concepts, and linguistic knowledge. Schematic knowledge represents categories of geometry word problems. For example, "find the area of a rectangle given its length and width" is categorized as a rectangular area problem (category 2a). Schematic knowledge includes lots of problem concepts that are used in geometry word problems, such as "rectangle", "given length", and "given width". Problem concept consists of linguistic knowledge needed for comprehending the linguistic descriptions of this problem concept.

Figure 1 is a tree structure for finding the area of a rectangle given its length and width. The tree contains the needed problem concepts and their linguistic knowledge. If learners are equipped with such cognitive knowledge, they should be able to comprehend problems of this type by producing their mental representation of these problems. This mental representation is known as the problem’s concept-attribute content, which will be discussed in the next section.

**< Insert Fig. 1>**

*3.2 Concept-attribute content of a problem*

A problem’s concept-attribute content is an internal mental representation of the problem after successful comprehension. This content can be represented as a tree structure that includes nodes of problem category, problem concepts, their attributes and values. Figure 2 shows the concept-attribute content module produced after the comprehension of Problem 1, which is categorized as “Find the area of a rectangle given its length and width.” The problem concepts include the concepts of rectangle, given length, given width, and area finding. The concept of length consists of a numeric value and a unit. For example, in this problem, the numeric value of the length of the rectangular object is one and the unit is kilometer, which means that the object is one kilometer long.

**< Insert Fig. 2>**

*3.3 Diagnosis of the comprehension level of geometry word problem*

Using the learner-initiating instruction strategy, the system can accept any geometry word problems entered by a learner. After comprehending a problem, the system provides simpler problem representations to improve the learner’s comprehension. At the next step, the system asks questions to check how well the learner comprehends the problem. For successful comprehension of a word problem, the learner needs to have the relevant cognitive knowledge, including linguistic and schematic knowledge. If a learner possesses the needed linguistic knowledge, he can understand the problem concepts of the words in the problem. If a learner understands the integrated meaning of all the concepts in the problem, he can identify the category of the problem. If the learner lacks the cognitive knowledge, he will not be able to comprehend the problem correctly.

The system asks diagnosis questions in terms of the problem’s concept-attribute content in order to check whether the learner is equipped with the needed cognitive knowledge for problem comprehension. First, the system asks the learner about the attributes and values of the concepts in the problem. If the learner answers correctly, it means that he has the linguistic knowledge to comprehend this problem. Next, depending on the problem category, the system asks the learner what information he needs in order to solve the problem. If the learner answers correctly, it means that he has the schematic knowledge and is able to ignore irrelevant information in the problem. For example, Problem 1 is an area-finding problem of a rectangle and the area is the product of the length and the width of the rectangle. Therefore, the learner should ignore irrelevant information in the problem and focus on the length and the width. Table 6 shows the diagnosis questions for Problem 1.

**< Insert Table 6>**

**4. System architecture**

LIM-G is composed of five components: input interface, cognitive knowledge base, concept retrieval module, representation generation module, and comprehension diagnosis module (Figure 3).

**< Insert Fig. 3>**

First of all, a learner enters a problem through the input interface (Figure 4). Then the concept retrieval module finds out major concepts of the problem and builds up the problem's concept-attribute content in the cognitive knowledge base. The representation generation module generates diagrammatic and telegraphic problem representations using the problem's concept-attribute content. The module is implemented with Scalable Vector Graphics (SVG), which is a language for describing two-dimensional graphics and graphical applications in XML (Lilley & Jackson, 2004). Finally, the comprehension diagnosis module asks the learner some questions in order to diagnose how well he comprehends the problem. The cognitive knowledge base and the concept retrieval module will be described in this section.

**< Insert Fig. 4>**

### *4.1 The knowledge representation of the cognitive knowledge base*

The cognitive knowledge base is used for comprehending word problems and is implemented with the InfoMap knowledge engineering tool provided by the Intelligent Agent System Lab, Institute of Information Science, Academia Sinica (Hsu, Wu, & Chen, 2001). InfoMap is an ontology-based system for knowledge representation and template matching. In this module, cognitive knowledge is organized as a hierarchy of nodes, which can be primarily divided into two classes: generic node and function node. In the cognitive knowledge base, generic nodes include several major types of nodes: problem-class schematic node, problem concept node, linguistic knowledge node, lexical node, and reference node (Figure 5).

**< Insert Fig. 5>**

### *4.2 Concept retrieval module*

The concept retrieval module retrieves the concepts of geometry word problems from the cognitive knowledge. The concepts in a geometry word problem are the prerequisite concepts a learner needs to know in order to comprehend the problem. The concept retrieval process follows several steps: problem concept template matching, frame matching of schematic knowledge of problem categories, and generating the concept-attribute content of the problem. The system first receives a geometry word problem from the input interface. Then, the template nodes matching all words in the problem will be triggered. The concepts of the problem will be determined after a series of triggered node firings. Moreover, the system looks for the problem-class schematic nodes matching most conditions of the fired problem concepts. Take Problem 1 for example. Its potential problem concept nodes and corresponding problem-class schematic nodes are listed in Table 7. According to both the concept matched rate and number of matched concepts, the target problem-class schematic node can be concluded as "*Find the area of a rectangle given its length and width*". Finally, the system identifies the template and the matched words to retrieve the concept's attributes and values, thus producing the problem's concept-attribute content that is needed for the representation generation module and the comprehension diagnosis module.

**< Insert Table 7>**

## 5. Experiment and discussion

An experiment is carried out to assess the degree to which LIM-G comprehends geometry word problems in Taiwan's elementary schools. Three levels of comprehension are differentiated: full comprehension, partial comprehension, and incomprehension. The following three conditions must be met in order to achieve full comprehension: (1) All the needed concepts of a problem are retrieved, (2) The category of a problem is correctly identified, and (3) The concept-attribute content of a problem is retrieved. When these three conditions are met, it means that the system fully comprehends the geometry word problem. If no condition is satisfied, incomprehension results. Any intermediate level of comprehension is called partial comprehension.

The geometry word problems in this experiment are gathered from five publishers in Taiwan (60 textbooks and reference books from the Ministry of Education, and publishers Kang Hsuan, Nani, Han Lin, and Senseio). All the elementary math textbooks and exercise books are standardized books that comply with the course objectives specified by the Ministry of Education of 1996. These books contain 282 geometry word problems presented as textual descriptions without any graphical illustration. Besides, these problems involve seven classes of geometric shapes, including squares, rectangles, circles, triangles, trapezoids, parallelograms, and sectors (Table 8).

As shown in Table 8, the rates of full and partial comprehension are respectively 84% and 16% for square problems, 90% and 10% for rectangle problems, 81% and 19% for circle problems, 92% and 8% for triangle problems, 85% and 15% for sector problems, 83% and 17% for trapezoid problems, and 100% and 0% for parallelogram problems. For all problems, the average rates of full and partial comprehension are 85% and 15% respectively. Besides, the rate of incomprehension is 0% for all kinds of problems. This result indicates that LIM-G so far can fully comprehend 85% of geometry word problems, with 15% partial comprehension and 0% incomprehension. Moreover, using the cognitive knowledge representation and frame-template structure in our system implementation, LIM-G can achieve such high rate of full comprehension with only 169 nodes. We believe these nodes cover most of the commonly used vocabulary in the geometry word problems in elementary schools in Taiwan.

**< Insert Table 8>**

All partially comprehensible problems are those which involve two different geometric shapes. For example: "*There is a rectangle 100m long and 60m wide. We dig a square pond with sides of 10m inside the rectangle. Please find the remaining area of this rectangle*." How to fully comprehend this kind of problems will be considered in our future work.

## 6. Potential applications and future work

Math word problems in mathematical education are used to develop students' skills in knowing when and how to apply their mathematical knowledge for approaching and solving problems in the real world (Burkhardt, 1994; Greer, 1997). But there is evidence that the math word problems used in school do not serve this objective well (Verschaffel & De Corte, 1997). By the end of elementary school, many students cannot apply their formal mathematical knowledge to solving real-word problems (Nesher, 1980); they do not have flexible access to heuristic and metacognitive strategies for attacking non-standard problems (De Corte, 1992; Verschaffel & De Corte, 1997); they only have a weak understanding of arithmetic operations as models of situations (Greer, 1992). Some researchers think that these problems occur because our classroom practice leads to unrealistic beliefs commonly held by many students. Every problem presented by the teacher or in the text book makes sense and is solvable; there is only one solution to every word problem; the solution can be obtained by performing one or more mathematical operations with the numbers in the problem, and almost certainly with all given numbers; the problem contains all the information needed by the solution (De Corte & Verschaffel, 1985; Schoenfeld, 1991). In order to address these issues, researchers suggest that students should also be asked to solve out-of-school problems, such as problems with extraneous and missing information (Verschaffel & De Corte, 1997). In LIM-G, when a learner enters an out-of-school problem, the system can diagnose the solvability of the problem, and whether there is missing or extraneous information. In a pilot study, we asked each of 18 students at fourth and fifth grades to pose ten problems for their peers to solve. Among the 180 problems they posed, 110 were standard, solvable problems and 70 were non-standard problems. Table 9 shows the results of understanding the 70 non-standard problems by LIM-G. The system correctly diagnosed if these problems had missing or extraneous information, or their goals were not specified.

**< Insert Table 9>**

For future work, we will add more functions to help learners learn and solve out-of-school problems. We will consider more problem representations that help learners comprehend word problems, such as semantic network and schema (Greeno, 1987). Many studies find these representations useful for students to do problem solving (Briars & Larkin, 1984; Kintsch & Greeno, 1985; Shalin & Bee, 1985). At this point, LIM-G is not able to fully comprehend about 15% of the geometry word problems in elementary schools. We will try to improve the mechanism of LIM-G in order to overcome this obstacle. Finally, we will extend LIM-G to cover others domains, such as time and algebra word problems by constructing the needed domain knowledge. Whether LIM-G is general enough to extend to these domains deserves further investigation.

## 7. Conclusion

In this study, a methodology of Learner-initiating Instruction Model (LIM-G) is proposed to help elementary students to comprehend geometry word problems. A learner can enter any geometry word problem he is interested in. After comprehending the problem, LIM-G provides telegraphic and diagrammatic problem representations to highlight the key features of the problem. Next the system asks the learner some questions in order to diagnose how well he comprehends the problem. If he is not able to identify any feature in the problem, the system assists the learner to focus on the features in the telegraphic and diagrammatic representations. LIM-G is an interactive tool, which can deal with standard and non-standard geometry word problems and guide the learner to understand the problems interactively.

The most powerful feature of LIM-G is its mechanism for comprehending geometry word problems. First, cognitive knowledge is used to extract the concepts from the problem and determine the category of the problem. Then LIM-G uses an ontology-based knowledge engineering tool, InfoMap, to represent the cognitive knowledge for comprehending word problems. Furthermore, the system can construct the problem's concept-attribute content with the frame-template structure. With the problem's concept-attribute content, the system can generate both telegraphic and diagrammatic representations and then ask the learner questions to diagnose his level of understanding.

We have conducted an empirical experiment to examine the ability of LIM-G to comprehend geometry word problems. The experiment uses the geometry word problems in Taiwan's elementary schools and achieves 85% full comprehension, 15% partial comprehension, and 0% incomprehension. Moreover, LIM-G can achieve this high rate of full comprehension with only 169 cognitive knowledge nodes. In future work, we will improve the mechanism of LIM-G in order to fully comprehend the difficult problems that are only partially comprehended by our current system.

An important goal of elementary education, which is often taken for granted, is to train students to solve real-life problems, which might be unsolvable, have missing or extraneous information, and almost never appear in a textbook or asked in the classroom. This goal can hardly be reached if textbooks and instructors use only problems that are always solvable, contain all needed and no extraneous information for deriving the solutions. If this goal is taken seriously, learners must be asked to solve non-standard, out-of-school problems. When they attempt to solve non-standard problems, they are likely to encounter difficulties and need help. Following an instructor-initiating strategy, most computer-assisted tutoring systems generally use a pre-designed database of problems and do not address the issue of understanding non-standard problems. In contrast, the LIM-G model, adopting a learner-initiating instruction strategy, is an important step in providing an intelligent tutor that assists learners to understand and solve non-standard problems. The greatest strength of LIM-G is its ability to comprehend standard and non-standard geometry word problems, since it can diagnose if the problem belongs to a well-defined category, or has extraneous or missing information. This ability is derived from a knowledge base that contains class-schematic, linguistic and semantic knowledge collected empirically from textbooks from various publishers.

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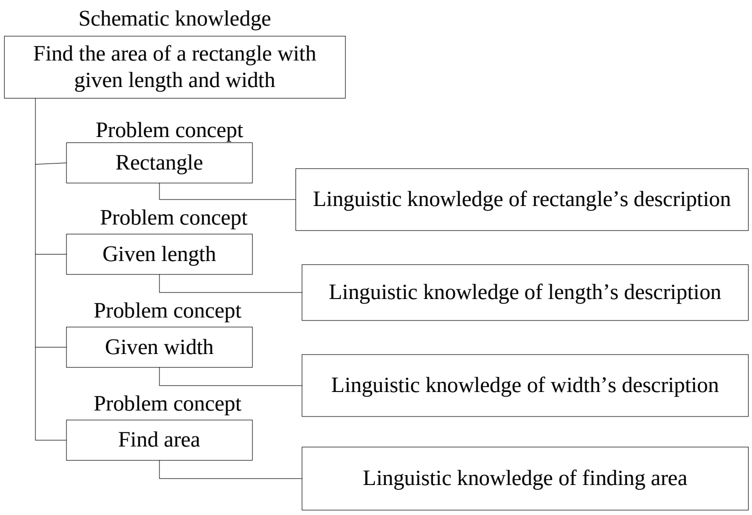



Fig. 1. Cognitive knowledge structure of a geometry word problem of category 2a

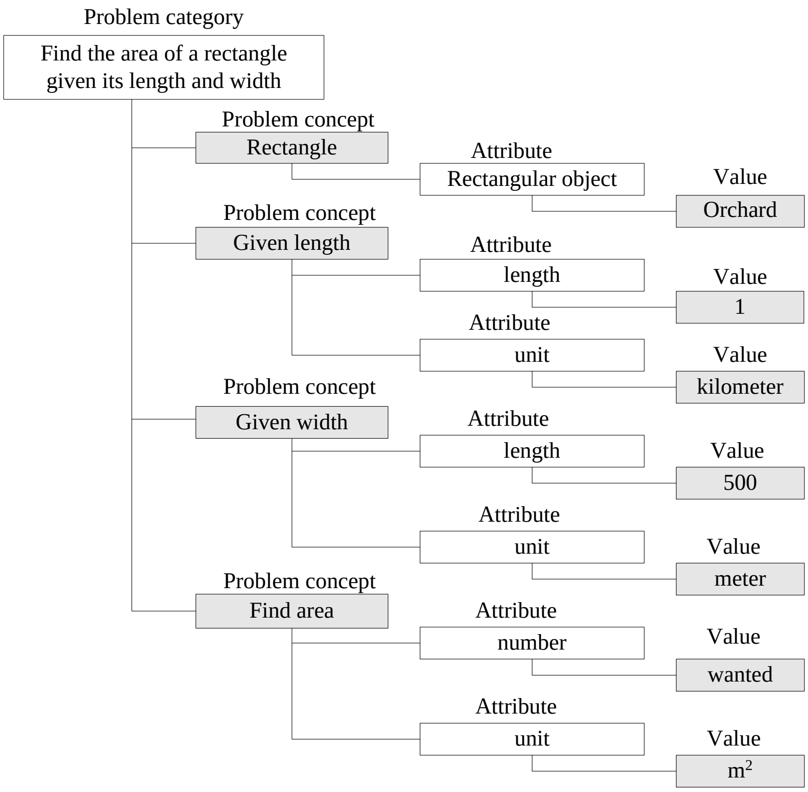



Fig. 2. The concept-attribute content of Problem 1

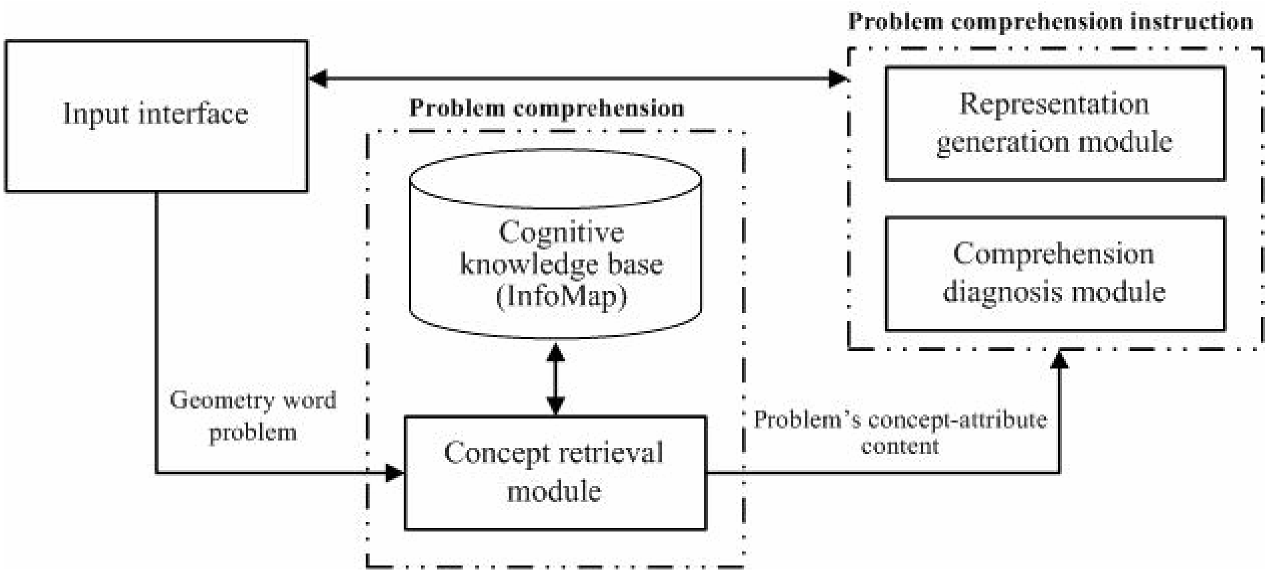



Fig. 3. The system architecture of LIM-G


Fig. 4. Input interface

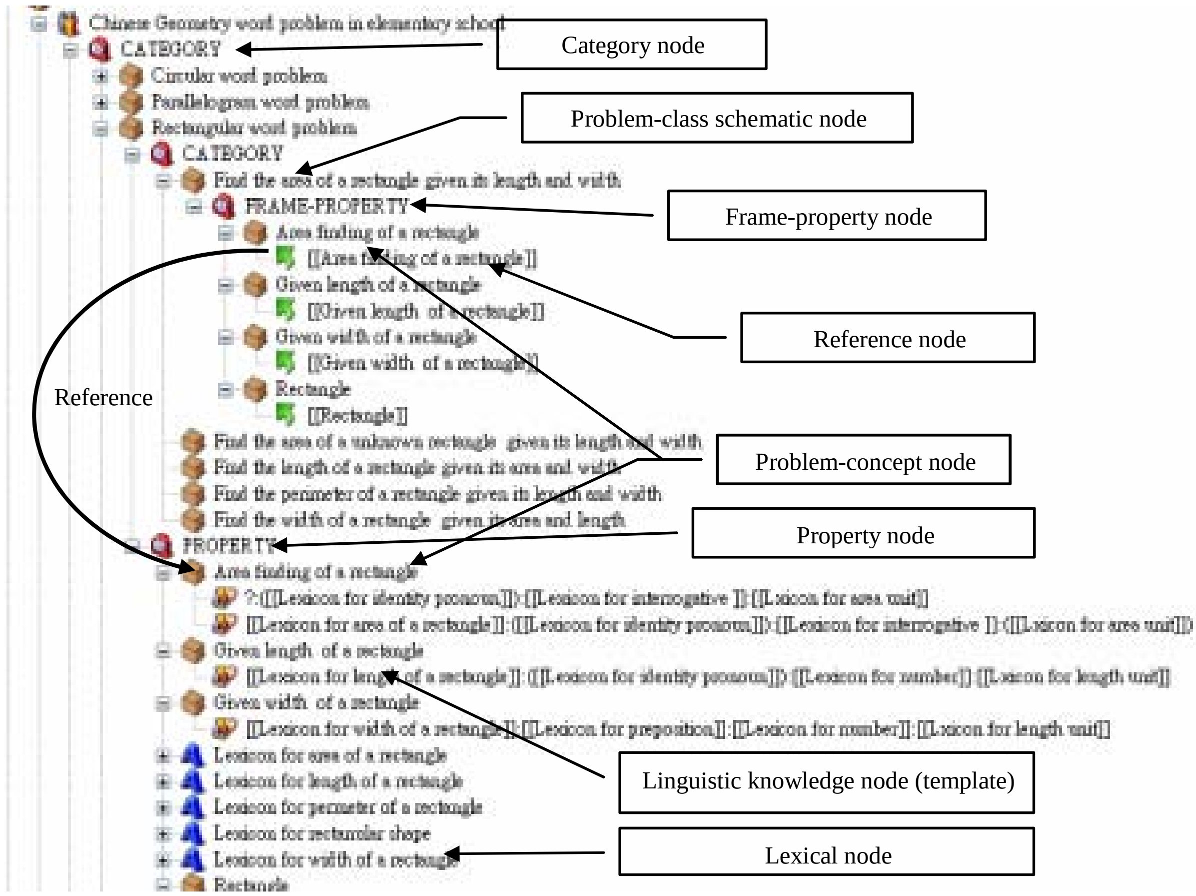



Fig. 5. The cognitive knowledge of cognitive knowledge base

Table 1
Five types of math word problems

| Math Word problem | Domain knowledge | Example |
|---|---|---|
| Arithmetic | Number | Jacob has 5 soda cans. Then Kathy gave him 3 soda cans. How many soda cans does Jacob have now? |
| Geometry | Geometry, number | There is a rectangular land with length 1km and width 500m. What is its area? |
| Time | Time, number | It takes two hours from Taichung to Taipei by train. Tom leaves Taichung at 11 AM by train. When will he arrive at Taipei? |
| Velocity | Velocity, time, number | The velocity of an ambulance is 50 miles per hour; how far is the hospital if it takes the ambulance one half of an hour to get there? |
| Algebra | Algebra, number | When added, four consecutive numbers have a sum of 18. What are the numbers? |

Table 2
The knowledge for comprehending Problem 1

| Knowledge | Knowledge content |
|---|---|
| Linguistic knowledge | Orchard can be a rectangle with length and width.<br>The area of this orchard in square meters can be computed. |
| Semantic knowledge | 1km equals to 1000m, area of rectangle = length * width |
| Schematic knowledge | This is an area problem of a rectangle with given length and width. |

Table 3
The distribution of Chinese geometry word problems in Taiwan's elementary school

| Grade | Grade 4 | Grade 5 | Grade 6 |
|---|---|---|---|
| Square | Square and rectangle (15) | Multiplication and division with fraction (8) | Land area (6)<br>Multiplication and division with decimal (2) |
| Rectangles | Square and rectangle (44) | Multiplication and division with fraction (6) | Land area (18)<br>Multiplication and division with decimal (15) |
| Triangles | None | Chapters of triangle (10) | Multiplication and division with decimal (2) |
| Parallelograms | None | Parallelograms (10) | Land area (1)<br>Multiplication and division with decimal (1) |
| Trapezoids | None | Trapezoids (9) | Multiplication and division with decimal (2) |
| Circle | None | Circle (106) | Multiplication and division with decimal (15) |
| Sector | None | Sector (12) | Land area (1) |

Note: In bracket is the number of problems for each shape in each grade.

Table 4
Categories of geometry word problems in Taiwan's elementary schools.

| Geometric shapes | Category |
|---|---|
| 1. Square (31) | 1a. Find the area of a square given the length of its sides<br>1b. Find the perimeter of a square given the length of its sides<br>1c. Find the area of a square given its perimeter<br>1d. Others for square |
| 2. Rectangle (83) | 2a. Find the area of a rectangle given its length and width<br>2b. Find the area of an unknown rectangle given its length and width<br>2c. Find the perimeter of a rectangle given its length and width<br>2d. Find the length of a rectangle given its area and width<br>2e. Find the width of a rectangle given its area and length<br>2f. Others for rectangle |
| 3. Circle (121) | 3a. Find the diameter of a circle given its radius<br>3b. Find the radius of a circle given its diameter<br>3c. Find the circumference of a circle given its diameter<br>3d. Find the circumference of a circle given its radius<br>3e. Find the area of a circle given its diameter<br>3f. Find the area of a circle given its radius<br>3g. Find the diameter of a circle given its circumference<br>3h. Find the radius of a circle given its circumference<br>3i. Find the area of a circle given its circumference<br>3j. Others for circle |
| 4. Triangle (12) | 4a. Find the area of a triangle given its base and height<br>4b. Find the base of a triangle given its area and height<br>4c. Others for triangle |
| 5. Sector (13) | 5a. Find the area ratio of a sector given its central angle<br>5b. Find the central angle of a sector given its area ratio<br>5c. Find the area of a sector given its radius and central angle<br>5d. Find the area of a sector given its radius and area ratio<br>5e. Find the circumference of a sector given its radius and area ratio<br>5f. Find the area of a sector given its diameter and area ratio<br>5g. Others for sector |
| 6. Trapezoid (11) | 6a. Find the area of a trapezoid given its base1, base2, and height<br>6b. Others for trapezoid |
| 7. Parallelogram (11) | 7a. Find the area of a parallelogram given its base and height<br>7b. Others for parallelogram |

Note: In bracket is the number of problems for the given type of shape.

Table 5
Three representations of a geometry word problem

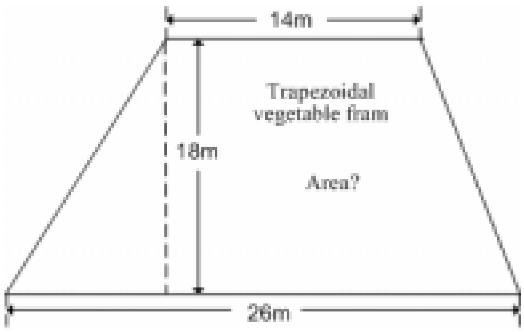
| | Example |
|---|---|
| Textual representation | Father reclaimed a trapezoidal vegetable farm on a piece of vacant land. Base1 and base2 are respectively 14m and 26m long, while the perpendicular distance between the bases is 18m high. What is the area of this farm? |
| Diagrammatic representation |   |
| Telegraphic representation | A trapezoidal vegetable farm,<br>base1: 14m, base2: 26m, height: 18m, find the area? |

Table 6
The diagnosis questions for Problem 1

| Cognitive knowledge | Problem concept | Attribute and value/Formula | Diagnosis question |
|---|---|---|---|
| Linguistic knowledge | Rectangle | Rectangular object: *orchard* | *1. What is the shape of the orchard?* |
| Linguistic knowledge | Given length of rectangle | The length of rectangle: *1km*<br>Unit of the length of rectangle: *kilometer* | *2. What is the length of the orchard?*<br>*3. What is the unit of the length of the orchard?* |
| Linguistic knowledge | Given width of rectangle | The width of rectangle: *500m.*<br>Unit of the width of a rectangle: *meter.* | *4. What is the width of the orchard?*<br>*5. What is the unit of the width of the orchard?* |
| Linguistic knowledge | Find the area of rectangle? | Unit of the area: s*quare meter.* | *6. Which attribute of the orchard is wanted?*<br>*7. What is the unit of the area?* |
| Schematic knowledge | Find the area of rectangle given its length and width. | Use the given length and width | *8. Which pieces of information are needed in order to solve the problem?* |
| Semantic knowledge | Area formula | Rectangular area = length* width | *9. How to compute the area of a rectangle from the length and width?* |

Table 7
The frame matching information of Problem 1

| Problem-class schematic node | Problem concept | | Concept matched rate | Matched concepts |
|---|---|---|---|---|
| Find the area of a rectangle given its length and width | 1. Rectangle<br>2. Length of a rectangle<br>3. Width of a rectangle<br>4. Find the area of a rectangle | [matched]<br>[matched]<br>[matched]<br>[matched] | 4/4 | 4 |
| Find the area of a rectangle given its length and width | 1. Length of a rectangle<br>2. Width of a rectangle<br>3. Find the area of a rectangle | [matched]<br>[matched]<br>[matched] | 3/3 | 3 |
| Find the perimeter of a rectangle given its length and width | 1. Rectangle<br>2. Length of a rectangle<br>3. Width of a rectangle<br>4. Find the perimeter of a rectangle | [matched]<br>[matched]<br>[matched]<br>[unmatched] | 3/4 | 3 |
| Find the length of a rectangle given its area and width | 1. Area of a rectangle<br>2. Width of a rectangle<br>3. Find the length of a rectangle | [unmatched]<br>[matched]<br>[unmatched] | 1/3 | 1 |

Table 8
The experimental result of comprehending geometry word problems from textbooks

| Shape used in problems | Number of problems of each shape | Full comprehension | Partial comprehension | Incomprehension |
|---|---|---|---|---|
| Square | 31 | 84% | 16% | 0% |
| Rectangle | 83 | 90% | 10% | 0% |
| Circle | 121 | 81% | 19% | 0% |
| Triangle | 12 | 92% | 8% | 0% |
| Sector | 13 | 85% | 15% | 0% |
| Trapezoid | 11 | 83% | 17% | 0% |
| Parallelogram | 11 | 100% | 0% | 0% |
| Total / Average | 282 | 85% | 15% | 0% |

Table 9
The experimental result of comprehending and diagnosing non-standard problems

| Diagnosis/<br>Out-of-school problem | Missing information | Extraneous information | Unsolvable | Accuracy |
|---|---|---|---|---|
| Missing information (27) | 27 | 0 | 0 | 100% |
| Extraneous information (28) | 0 | 28 | 0 | 100% |
| Goal not specified (15) | 0 | 0 | 15 | 100% |